Modern Physics

Do <u>two</u> of the following three problems, each on a separate sheet (or sheets). Attach each set to a provided cover sheet with your name, subject, and problem number.

Problem 1

Consider an electron in a uniform magnetic field in the positive z-direction:

$$H = \mu_0 B \sigma_z$$

with

$$\sigma_z = \hbar \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

where μ_0 is the magnetic moment of the electron and B is the magnitude of the magnetic field.

(a) Use the outcome of a Stern-Gerlach experiment to explain that the $S_x = \pm\frac{1}{2}$ states in the S_z basis can be expressed as

$$|\pm x\rangle = \frac{1}{\sqrt{2}} \left(|+z\rangle \pm |-z\rangle \right).$$

(b) The result of a measurement has shown that the electron spin is along the positive x-direction at time $t = 0$. For $t > 0$ compute quantum mechanically the probability of finding the electron in one of the states $|+x\rangle$, $|-x\rangle$, or $|+z\rangle$.

Problem 2

Consider a quantum mechanical system that has only two energy eigenstates: $|1\rangle$ and $|2\rangle$. Besides the energy there are three other observables, called X, Y, Z. The energy eigenstates $|1\rangle$ and $|2\rangle$ are normalized but not necessarily eigenstates of X, Y, or Z.

Determine as many of the eigenvalues of X, Y, and Z as possible on the basis of the following "data sets". (Warning: one data set is unphysical.)

(a) $\langle 2| X |2\rangle = \frac{1}{3}$, $\langle 2| X^2 |2\rangle = \frac{1}{9}$.

(b) $\langle 2| Y |2\rangle = \frac{1}{3}$, $\langle 2| Y^2 |2\rangle = \frac{1}{10}$.

(c) $\langle 2| Z |2\rangle = 1$, $\langle 2| Z^2 |2\rangle = \frac{10}{9}$, $\langle 2| Z^3 |2\rangle = \frac{4}{3}$.

Problem 3

A particle is described by the wave function

$$\psi(x,t) = A_0\psi_0(x)e^{-\frac{i}{\hbar}E_0 t} + A_1\psi_1(x)e^{-\frac{i}{\hbar}E_1 t},$$

where $\psi_i(x)$ are normalized solutions to the stationary Schrödinger equation with energy E_i $(i = 0, 1)$. A_i are some constants such that $|A_0|^2 + |A_1|^2 = 1$. At time t_1 the energy is measured and found to be equal to E_1. If at time $t_2 > t_1$ the energy is measured again, what is the probability that the outcome of the measurement at t_2 again yields E_1?