Quantum Mechanics

Do two of the following three problems, each on a separate sheet (or sheets). Staple together the sheets for each problem, if using multiple sheets, but do not staple all problems together. Write at the top of the first sheet of each problem your name, subject, and problem number.

Problem 1

Suppose an electron is in a state described by the wave function

$$\psi = \frac{1}{\sqrt{4\pi}} \left(e^{-\phi} \sin\theta + \cos\theta \right) g(r)$$

with

$$\int_0^\infty |g(r)|^2 \, r^2 dr = 1.$$

(a) What are the possible results of a measurement of the z-component of the angular momentum, L_z, of the electron in this state?

(b) What is the probability of obtaining each of the possible values obtained in part **(a)**?

(c) What is the expectation value of L_z in this state?

Problem 2

A quantum system is characterized by a discrete spectrum whose eigenstates are $|\psi_1\rangle$ and $|\psi_2\rangle$ with corresponding energies $E_1 = 0$ and $E_2 = M$.

(a) Write the Hamiltonian $\hat{H}_0$ of the system in the basis of these two states as a 2×2 matrix.

(b) Now, consider a perturbation written, in the same basis, as the matrix

$$\hat{V} = \begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix},$$

with $m = \lambda M$, where $\lambda \ll 1$. The new Hamiltonian is $\hat{H} = \hat{H}_0 + \hat{V}$. Using perturbation theory, calculate the first- and second-order corrections to the eigenvalues of $\hat{H}_0$.

Problem 3

A one-dimensional potential barrier has the form $V(x) = V_0\delta(x)$, where $V_0 > 0$ and $\delta(x)$ is the Dirac delta function. Calculate the transmission and reflection coefficients through the barrier for a particle of mass m .