Quantum Mechanics

Do two of the following three problems, each on a separate sheet (or sheets). Staple together the sheets for each problem, if using multiple sheets, but do not staple all problems together. Write at the top of the first sheet of each problem your name, subject, and problem number.

Problem 1

Consider an electron in a uniform magnetic field pointing along the z-direction. The electron spin at time t_0 points along the positive y-axis.

a) What are the expectation values of the magnetic polarization along the x-axis, y-axis, and z-axis, $\langle 2S_x \rangle$, $\langle 2S_y \rangle$ and $\langle 2S_z \rangle$, respectively, for $t > t_0$?

b) What type of phenomenon is described by the results you found in part **a)**? Explain.

For reference, the spin matrices are

$$S_x = \frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad S_y = \frac{\hbar}{2}\begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}, \quad S_z = \frac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Problem 2

Consider a quantum system described by the Hamiltonian matrix

$$\hat{\mathcal{H}} = U \begin{bmatrix} 3 & 0 & -2i \\ 0 & 3 & 0 \\ 2i & 0 & 3 \end{bmatrix}, \tag{1}$$

where U is a real constant with units of energy.

a) (*2 pts.*) Verify that the Hamiltonian (1) is **Hermitian**.

b) (*1 pt.*) Without calculating anything – **How many possible energy levels** can there be for the system described by the Hamiltonian (1)? (You may assume that the energies are nondegenerate.)

c) (*4 pts.*) Find **all possible outcomes** of measuring the **energy** using the Hamiltonian (1).

d) (*3 pts.*) Find the **properly normalized ground state** of the Hamiltonian (1) (the state vector corresponding to the lowest possible energy).

Problem 3

Use the variational method to estimate the ground state energy E_0 of the potential

$$V(z) = \begin{cases} mgz, & z \geq 0, \\ \infty, & z < 0. \end{cases}$$

Use the (normalized) trial wave function

$$\psi_t(z) = \left(\frac{128\alpha^3}{\pi}\right)^{1/4} z e^{-\alpha z^2}$$

which satisfies the boundary condition at $z = 0$.