Quantum Mechanics

Do <u>*two*</u> *of the following three problems, each on a separate sheet (or sheets). Staple together the sheets for each problem, if using multiple sheets, but do not staple all problems together. Write at the top of the first sheet of each problem your name, subject, and problem number.*

Problem 1

A 1-dimensional quantum mechanical system for $x \in]-\infty, +\infty[$ is described by the following Hamiltonian:

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + bx^4$$

and we assume a trial wave function of the form:

$$\phi(x) = Ae^{-\lambda^2 x^2}.$$

Answer the following questions for this system:

a) Provide at least one reason why the trial wave function is not a bad choice for this Hamiltonian.

b) Show that the constant can be expressed as $|A|^2 = \sqrt{\frac{2\lambda^2}{\pi}}$. (Hint: Polar coordinates.)

c) Determine the value of λ that leads to the best estimate of the ground state energy of this system and compute the corresponding ground state energy.

The following integrals may be useful and can be used without proof:

$$\int_{-\infty}^{+\infty} x^2 \cdot e^{-2\lambda^2 x^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{8\lambda^6}}$$
$$\int_{-\infty}^{+\infty} x^4 \cdot e^{-2\lambda^2 x^2} dx = \frac{3}{4}\sqrt{\frac{\pi}{32\lambda^{10}}}$$

Problem 2

A quantum system has 3 stationary energy levels: the state $|1>$ has energy ε; the state $|2>$ has energy 3ε; and the state $|3>$ has energy 9ε. If the system is initially prepared in the state

$$|\psi_0> = \frac{1}{\sqrt{6}}|1> + \sqrt{\frac{2}{3}}|2> - \frac{1}{\sqrt{6}}|3>$$

at time $t = 0$, then **find the state at the later time** $t = \frac{\pi}{2}\frac{\hbar}{\varepsilon}$.

Problem 3

An electron is prepared in a three-dimensional state, with wave function given by

$$\psi(r, \theta, \phi) = R(r) \cos^2 \phi,$$

where $R(r)$ is a radial wave function and ϕ is the azimuthal angle in spherical coordinates. **What are the possible outcomes of measuring the z-component of angular momentum l_z** (also called the magnetic quantum number m)? What are **the respective probabilities of these measurements**? You may find the following identities useful:

$$\begin{aligned} \cos 2\theta &= \cos^2 \theta - \sin^2 \theta, \\ e^{i\theta} &= \cos \theta + i \sin \theta. \end{aligned}$$