Statistical Physics

January 21, 2017

Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

Assume you have three particles and 4 degenerate (i.e. same energy) microstates. Assume that the partition function is described simply by the total number of distinct system states.

a.) Assuming that the particles are distinguishable and double or triple occupancy is allowed, determine the partition function of that system?

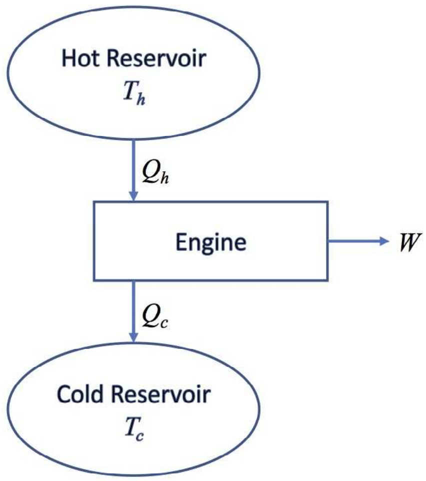
b.) Assuming that the particles are indistinguishable and double or triple occupancy is allowed, determine the partition function of that system?

c.) Assuming that the particles are indistinguishable and double or triple occupancy is not allowed, determine the partition function of that system?

d.) What are the probabilities of finding exactly two particles in the same microstate for all three cases above?

Problem 2

Heat Engine: The diagram shows the energy flow in a heat engine.



a.) State the definition of the efficiency of this heat engine in terms of the heat Q_h absorbed from the hot reservoir and the heat Q_c injected into the cold reservoir. Do not assume that the engine is reversible.

b.) Based upon entropy considerations, derive an inequality between the efficiency of the engine and the temperatures of the reservoirs.

Now replace the reservoirs with two identical finite bodies, each characterized by a heat capacity at constant pressure, C, which is independent of temperature. The two bodies remain at constant pressure and undergo no phase change. Initially the temperature of the hot body is T_1 and that of the cold body is T_2; finally, as a result of the operation of the heat engine, both bodies will come to a common final temperature T_f.

c.) What is the amount of work done by the engine, as the bodies go from their initial temperatures to the final common temperature? Do not assume the engine is operated reversibly. Express your answer in terms of C, T_1, T_2, and T_f.

d.) Use entropy considerations to derive an inequality relating T_f to the initial temperatures T_1 and T_2.

e.) For given initial temperatures T_1 and T_2, what is the maximum amount of work that could ever be obtained by any engine operating between these two bodies?

Problem 3

a.) A system has two states of energies $-\epsilon$ and 2ϵ. What is the probability of observing it in the higher energy state at temperature T?

b.) Consider a classical diatomic gas consisting of N particles (i.e., $2N$ atoms forming N molecules) whose atoms are trapped on a layer of material such that the atoms can only move in two dimensions. What is the heat capacity at constant volume of this gas? You may assume that $k_B T \ll \hbar\sqrt{\frac{k}{m}}$ where m is the mass of each atom and k is the Hooke's law constant associated with vibrational motion.