Thermodynamics

Do two of the following three problems, each on a separate sheet (or sheets). Attach each set to a provided cover sheet with your name, subject, and problem number.

Problem 1

Consider the nucleus of ^{4}He as a non-relativistic Fermi gas of four, spin-$\frac{1}{2}$ nucleons (two protons and two neutrons) with a density of 0.18 nucleons per fm^3. Assume that the only energy is kinetic (that is, ignore any electromagnetic interaction between the protons) and that protons and neutrons have the same mass, $m_N = 0.94\,\mathrm{GeV}/c^2$, so that they can be considered as two states of the particle called the nucleon.

1. What is the degeneracy factor of the system? (1 point)
2. Calculate the Fermi energy ε_F. (6 points)
3. Examine whether the system is fully degenerate at room temperature. (1 point)
4. Based on the previous results, justify the use of non-relativistic equations. (2 points)

Useful constants: $h = 4.14 \times 10^{-15}\,\mathrm{eV\,s}$; $k = 8.62 \times 10^{-5}\,\mathrm{eV\,K^{-1}}$

Problem 2

The number of microstates available to an Einstein solid of N oscillators with q units of energy at low temperatures is:

$$\Omega(N, q) = \left(\frac{eN}{q}\right)^q,$$

where $q \ll N$.

a) If ε is the size of an energy unit, such that $U = q\varepsilon$, derive an expression for U in terms of T.

b) Derive an expression for the heat capacity at constant volume for such a system.

c) Provide a qualitative sketch of the temperature dependence of the heat capacity at low temperatures.

Problem 3

1. What is the thermodynamic definition of temperature? (2 points)
   Hint: the correct answer is not: "Take boiling water and freezing water and divide some difference by 100."

2. Consider a diatomic gas. Qualitatively, sketch a plot of the specific heat $c_V = \frac{\partial E}{\partial T}$ per molecule as a function of the temperature T. Clearly (i.e. quantitatively) label the limits $T \to 0$ and $T \to \infty$ as well as any plateaus should they appear. (4 points)

3. The temperature of a hypothetical fixed star is $T_s = 6000\,\mathrm{K}$. The radius of the star is $R_s = 1$ million km. The star is orbited by a fast spinning planet without an atmosphere and the radius of the orbit of the planet around the fixed star is $r_p = 100$ million km. The radius of the planet is $R_P = 1000\,\mathrm{km}$. In equilibrium (steady state), what is the surface temperature on the planet? (4 points)