Thermodynamics

Do two of the following three problems, each on a separate sheet (or sheets). Attach each set to a provided cover sheet with your name, subject, and problem number.

Problem 1

1. Consider N massless noninteracting spin $\frac{1}{2}$ particles in a three-dimensional box of volume V at zero temperature

   (a) find the Fermi energy as a function of N and V (3 points)

   (b) find the pressure (2 points)

2. The surface temperature of the sun is about $T = 6000$ K. Its radius is about 7×10^8 m. Consider a planet orbiting the sun at a distance of about 10^{11} m. Let's call that planet Venus. Treat both the sun and the planet as an ideal black body. Neglecting any possible greenhouse effects and assuming that the planet rotates fast enough that its surface temperature is uniform, what is the surface temperature of that planet in static equilibrium? (5 points)
   Hint: The law of Stefan and Boltzmann states that the total power radiated per unit surface area of a black body across all wavelengths per unit time equals σT^4, where $\sigma = 5.67 \times 10^{-8} \frac{\mathrm{W}}{\mathrm{m}^2\mathrm{K}^4}$.s

Problem 2

The isothermal compressibility $\kappa_T = \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_T$ of metals is well described by the free-electron model (an ideal gas of electrons). For metals, we find that the average thermodynamic energy is $E = \frac{3}{5}N\varepsilon_F$, where the Fermi energy is $\varepsilon_F = AV^{-2/3}$ with $A = \left(\frac{3}{8\pi}\right)^{2/3}\frac{h^2}{2m_e}N^{2/3}$.

a. Show that $P = a\frac{E}{V}$ and determine the constant a.

b. Use the results of part **a** to show that $\left(\frac{\partial V}{\partial P}\right)_T = b\frac{P}{V}$ and determine b.

c. Use the results of part **a** to show that $P = c\frac{N}{V}\varepsilon_F$ and determine c.

d. Use the results to determine κ_T for metals in terms of ε_F and $n = \frac{N}{V}$.

Problem 3

An ideal Carnot refrigerator is used to cool a food storage locker that has a constant heat capacity of C. The external heat reservoir is maintained at a constant temperature. Initially, the locker had the same temperature as the heat reservoir. The exterior walls of the locker do not conduct heat. Find the amount of work required to cool the locker from its initial temperature of T_1 to a final temperature of T_2.